Rendiconto Scientifico dell'attività del Consorzio Interuniversitario per l'Alta Formazione in Matematica (CIAFM) per il 2011

1 - Elenco dei Corsi 2011

2 - Partecipanti ai Corsi di Matematica di Perugia (6 corsi)

3 - Partecipanti ai Corso di Matematica di Cortona (2 corsi)

4 - Elenco dei partecipanti ai singoli Corsi - Perugia

5 - Programmi dei Corsi di Perugia e Cortona 2011

Rendiconto scientifico dell'attività del CIAFM per il 2011

Nel 2011 il Consorizio in collaborazione con la Scuola Matematica Interuniversitaria ha organizzato corsi di base per laureandi e giovani laureati nella sede di Perugia e corsi più avanzati, di avviamento alla ricerca a Cortona.

1 - ELENCO DEI CORSI 2011

***PERUGIA*: (31 luglio – 2 settembre 2011)**

Insegnamenti

ALGEBRA	Nikolai Vavilov	(Univ. San Pietroburgo)
ANALISI COMPLESSA	Klas Diederich	(Univ. Wuppertal)
ANALISI FUNZIONALE	Yuli Eidelman Vitali Milman	(Univ. Tel Aviv) (Univ. Tel Aviv)
EQUAZIONI DIFFERENZIALI DELLAFISICA MATEMATICA	Russel Johnson Carlo D. Pagani	(Univ. Firenze) (Politecnico Milano
GEOMETRIA ALGEBRICA	Enrique Arrondo	(Univ.Compl.Madrid)
PROBABILITA'	Sandy Zabell	(Northwestern Univ.)

CORTONA

Cortona I: 3 luglio – 16 luglio 2011

STOCHASTIC NUMERICAL METHODS OF FINANCE

Eckhard Platen - University of Technology Sydney, Australia
Wolfgang Runggaldier – Università di Padova

Cortona II: 21 agosto – 2 settembre 2011

HYPERBOLIC SYSTEMS OF CONSERVATION LAWS

Stefano Bianchini - SISSA Trieste
Constantine M. Dafermos - Brown University

2 – PARTECIPANTI AL CORSO ESTIVO DI MATEMATICA – PERUGIA (6 corsi)
31 luglio – 2 settembre 2011

<u>Studenti Italiani</u>
Domande: 55
Studenti ammessi: 50
Partecipanti effettivi:36

ARICI Francesca
BALDISSERRI Agnese
BONFANTI Matteo A.
BRANCACCIO Giulia
CAROCCIA Marco
CASSESE Daniele
CESARO Andrea
CIRRITO Vincenzo A.
COVATO Elisa
CRISMALE Vito
DE FALCO Vittorio
DOLCE
D'ONOFRIO Giuseppe
GALLETTI Federico
GIANGRASSO Giuseppina
LUPO Salvatore
LUSSO Lorena E.
MACCHIA Antonio
MANINI Maria Grazia
MARZIONI Simone
MASTROSTEFANO Stefano
MONDELLO Ilaria
MONGUZZI Alessandro
NALDI Simone
NEGRO Giuseppe
PANICO Michele
PORCELLO Giovanni
RIZZI Matteo
SACCO Armando
SANTINI Alberto Maria
SCALISE Jacopo V.
STRAZZANTI Francesco
TEALDI Lucia
TRANQUILLI Giorgia
VALLE Cristina
VANNACCI Matteo

Studenti stranieri

Domande: 43
Studenti ammessi: 35
Partecipanti effettivi:28

BATALKIN KKirill
BROZEK Marcin
CARRERE Cecile
CASTEJON DIAZ Hector
CRAWFORD Jonathan K.
CYGAN Wojciech
DRIMBE Daniel
ERCEG Marko
GLITIA Dana-Debora

HOWE Sean
KIRSHTEIN Arkadz
LESESVRE Didier
LIN Hsueh-Yung
MISUR Maria
MUSIAL Wojciech
NITULESCU Anca
PEGON Paul
SHCHEGOLEV Alexander
STAWSKA Anna
SWIDERSKI Grzegorz
TAYLAN Demet
THOMAS David R.
UGURLU Ozlem
VASQUEZ Scott
VLAD Emanuel
ZAKHAREVICH Valentin
ZELKO Ioana
ZWOLENSKI Pawel

3 –PARTECIPANTI AL CORSO ESTIVO DI MATEMATICA – CORTONA (2 CORSI)

STOCHASTIC NUMERICAL METHODS OF FINANCE

Partecipanti Italiani

Domande :24
Ammessi : 20
Partecipanti effettivi:15 + 1 uditore

BIANCHI Daniele Bocconi,Milano

BONACINA Fausto	Milano Bicocca
BONGIORNO Enea	Milano
CIOCIOA Giuseppe	Foggia
COSSO Andrea	Milano Politecnico
GENTILE Maria	Napoli
MAURO Paolo	Catania
MERCURI Lorenzo	Politecnica delle Marche
OLIVA Immacolata	Basilicata
PAGLIARANI Stefano	Bologna
PERI Ilaria	Milano Bicocca
POTENTE Gianluigi	Padova
REGOLI Daniele	Bologna
SALVI Giovanni	Roma "La Sapienza"
SANTOMAURO Giuseppe	Salerno
AGROSI Giancarlo	Salento (uditore)

<u>Partecipanti stranieri</u>

Domande : 7
Ammessi : 6
Partecipanti effettivi : 7 compreso 1 uditore

DRYL Monika	Bialystok
HAO Ji *	Beijing Technology
HITAJ Asmerilda *	Milano Bicocca
MAJEWSKI Adam	Gdansk
ODZIJEWICZ Tatiana	Bialystok
WANG Hao *	Firenze
SIYOU Romuald (uditore) *	Milano Cattolica

* Questi partecipanti sono studenti in Italia

<u>Cortona II: 21 agosto – 2 settembre 2011</u>

HYPERBOLIC SYSTEMS OF CONSERVATION LAWS

<u>Studenti Italiani</u>

Domande : 10
Ammessi : 9
Partecipanti effettivi : 7

CAPRIANI Giuseppe Maria	Palermo
DI RUVO Lorenzo	Napoli

FISCELLA Alessio	Perugia
IANNUZZI Rosa	Salerno
MAURIZI Amelio	L'Aquila
MAURO Jmmy Alfonso	Napoli e Pisa
VECCHI Eugenio	Bologna

<u>Studenti Stranieri</u>

Domande : 11
Ammessi : 10
Partecipanti effettivi : 6

DWIVEDI Shivanand	Indian Institute of Technology
MALOGROSZ Marcin	Warsaw
POGODAEV Nikolay	Irktusk State
SVIRIDOVA Elena	Voronezh
SVIRIDOVA Evgeniia Alexandrovna	Voronezh
YU LEI	Shangai Jiao Tong

<u>4 – PARTECIPANTI PER SINGOLI CORSI AL CORSO ESTIVI DI PERUGIA (6 CORSI)</u>

July 31 – September 2, 2011

<u>Algebra</u> – (30)

<u>Studenti italiani</u>

BALDISSERRI Agnese	Bologna
BONFANTI Matteo Alfonso	Milano Bicocca
CESARO Andrea	Milano Bicocca
COVATO Elisa	Catania
DOLCE Francesco	Palermo
GALLETTI Federico	Firenze
GIANGRASSO Giuseppina	Palermo
MACCHIA Antonio	Bari
MANINI Maria Grazia	Perugia
MARZIONI Simone	Bologna
NALDI Simone	Firenze
PANICO Michele	Perugia
STRAZZANTI Francesco	Catania
VANNACCI Matteo	Firenze

Studenti stranieri

CASTEJON DIAZ Hector	UPC, Spagna
CRAWFORD Jonathan Keith	Warwick
GLITIA Dana-Debora	Babes-Bolyai
HOWE Sean	Arizona State
LESESVRE Didier	ENS Cachan
MUSIAL Wojciech	MIT
NITULESCU Anca	Bucharest
PEGON Paul	ENS Cachan
SHCHEGOLEV Alexander	St.Petersburg
SWIDERSKI Grzegorz	Wroclaw
TAYLAN Demet	Bozok
THOMAS Dvid R.	MIT
UGURLU Ozlem	Dokuz Eylul
VASQUEZ Scott	MIT
VLAD Emanuel	Bucharest
ZAKHAREVICH Valentin	Polytech.Inst. NYU Univ.

Analisi Complessa (11)

Studenti italiani

LUSSO Eleonora	Cagliari
MONGUZZI Alessandro	Milano Bicocca
PORCELLIO Giovanni	Palermo
RIZZI Matteo	Milano Bicocca
TRANQUILLI Giorgia	Cagliari

Studenti stranieri

BROZEK Marcin	Karol Adamiecki University – Silesian
DRIMBE Daniel	Bucharest
LESESVRE Didier	ENS Cachan
LIN Hsueh-Yung	Ecole Normale Supérieure Lyon
PEGON Paul	ENS Cachan
STAWSKA Anna	Silesian

Analisi Funzionale (30)

Studenti italiani

ARICI Francesca	Cattolica Sacro Cuore
BRANCACCIO Giulia	L. Bocconi Milano
CAROCCIA Marco	Firenze
CASSESE Daniele	Siena
CIRRITO Vincenzo Alberto	Catania
CRISMALE Vito	Bari

D'ONOFRIO Giuseppe	Napoli
DE FALCO Vittorio	Napoli
LUPO Salvatore	Palermo
MASTROSTEFANO Stefano	Cassino
MONDELLO Ilaria	Milano Bicocca
MONGUZZI Alessandro	Milano Bicocca
NEGRO Giuseppe	Bari
PORCELLO Giovanni	Palermo
RIZZI Matteo	Milano Bicocca
SANTINI Alberto Maria	Catania
TEALDI Lucia	Milano Statale
TRANQUILLI Giorgia	Cagliari
VALLE Cristina	Torino

Studenti stranieri

CARRERE Cecile	ENS, Cachan
CYGAN Wojciech	Wroclaw
DRIMBE Daniel	Bucharest
ERCEG Marco	Zagreb
KIRSHTEIN Arkadz	Belarusian State
MISUR Marin	Zagreb
MUSIAL Wojciech	MIT
THOMAS David R.	MIT
VASQUEZ Scott	MIT
VLAD Emanuel	Bucharest
ZELKO Iana	MIT

Equazioni Differenziali della Fisica Matematica(18)

Studenti italiani

CAROCCIA Marco	Firenze
CRISMALE Vito	Bari
DE FALCO Vittorio	Napoli
LUPO Salvatore	Palermo
MANINI Maria Grazia	Perugia
MASTROSTEFANO Stefano	Cassino
NEGRO Giuseppe	Bari
PANICO Michele	Perugia
SACCO Armando	Roma Tre
SCALISE Jacopo Vittorio	Milano Bicocca
TEALDI Lucia	Milano Statale

Studenti stranieri

CARRERE Cecile	ENS Cachan
CYGAN Wojciech	Milano Statale
ERCEG Marco	Zagreb
KIRSHTEIN Arkadz	Belarusian State
MISUR Marin	Zagreb
ZELKO Iana	MIT
ZWOLENSKI Pawel	Silesian Katowice

Geometria Algebrica (28)

Studenti italiani

ARICI Francesca	Cattolica Sacro Cuore
BALDISSERRI Agnese	Bologna
BONFANTI Matteo Alfonso	Milano Bicocca
CESARO Andrea	Milano Bicocca
COVATO Elisa	Catania
DOLCE Francesco	Palermo

GALLETTI Federico	Firenze
GIANGRASSO Giuseppina	Palermo
LUSSO Lorena Eleonora	Cagliari
MACCHIA Antonio	Bari
MARZIONI Simone	Bologna
MONDELLO Ilaria	Milano Bicocca
NALDI Simone	Firenze
SCALISE Jacopo Vittorio	Milano Bicocca
STRAZZANTI Francesco	Catania
VALLE Cristina	Torino
VANNACCI Matteo	Firenze

<u>Studenti stranieri</u>

BATALKIN Kirill	St.Petersburg
CASTEJON DIAZ Herctor	UPC, Spagna
CRAWFORD Jonathan Keith	Warwick
GLITIA Dana-Debora	Babes-Bolyai
LIN Hsueh-Yung	Ecole Normale Supérieure Lyon
NITULESCU Anca	Bucharest
SHCHEGOLEV Alexander	St.Petersburg
SWIDERSKI Grzegorz	Wroclaw
TAYLAN Demet	Bozok
UGURLU Ozlem	Dokuz Eylul
ZAKHAREVIC Valentin	Polytech.Inst. NYU Univ.

<u>Probabilità (10)</u>

<u>Studenti italiani</u>

BRANCACCIO Giulia	L. Bocconi Milano
CASSESE Daniele	Siena
CIRRITO Vincenzo Alberto	Catania
D’ONOFRIO Giuseppe	Napoli
SACCO Armando	Roma Tre
SANTINI Alberto Maria	Catania

<u>Studenti stranieri</u>

BROZEK Marcin	Karol Adamiecki University – Silesian
HOWE Sean	Arizona State
STAWSKA Anna	Silesian
ZWOLENSKI Pawel	Silesian Katowice

à

5 – PROGRAMMI DEI CORSI ESTIVI DI CORTONA E PERUGIA 2011

Cortona 3 luglio – 16 luglio 2011

STOCHASTIC NUMERICAL METHODS OF FINANCE

Docente : Professor Eckhard Platen University of Technology Sydney - Eckhard.Platen@uts.edu.au

Objectives:

The aim of this course is to present various numerical methods used in modern Quantitative Finance. It develops further mathematical concepts, techniques and intuition necessary for modern financial modelling, derivative pricing and risk management. This subject provides the foundations for a sufficiently rigorous understanding of advanced numerical and statistical methods in finance. Emphasis will be laid on developing skills that allow students deal with numerical questions related to models involving stochastic differential equations for pricing and hedging complex financial products. Questions of numerical stability and convergence will be discussed in detail.

Reference Book:

The course will be based on the 2010 Springer book:
"***Numerical Solution of Stochastic Differential Equations with Jumps in Finance***"
by Platen and Bruti-Liberati, Springer Verlag, ISBN 0172-4568; ISBN 978-3-642-12057

The chapters covered and a brief description of their content are listed below:

Chapter	Summary
Ch 4: Stochastic Expansions	Presents stochastic Taylor expansions and their application.
Ch 5-8: Scenario Simulation	Introduces strong discrete time approximations for stochastic differential equations for the application in scenario simulation. Jump diffusions are covered.
Ch 11-14: Monte Carlo Simulation	Presents modern techniques for Monte Carlo simulation of stochastic differential equations in finance.
Ch 16: Variance Reduction Techniques	Describes a range of powerful methods that permit significant variance reductions in Monte Carlo simulation.

Ch 14: Numerical Stability	The propagation of errors is analysed and stability regions are discussed.
Topics that can be presented by participants:	
Ch 17: Trees and Markov Chains	A convergence result for tree based methods is presented. The advantages and disadvantages of tree methods are analysed.
Ch 17: Partial Differential Equations	Describes the relationship of finite difference methods with other numerical methods for partial differential equations.
Ch 2: Exact Simulation of Solutions of SDEs	Describes the exact and almost exact simulation of solutions of stochastic differential equations.
Ch 10: Filtering in Finance	Introduces into the filtering of hidden quantities that are modelled by Markov chains and diffusions.

Comments:

The course will be presented in the above sequence of topics. The detailed scheduling of the lectures will depend on the exact number of lectures, their length and the feedback from the participants. Exercises will be given and solutions will be made available. The presented numerical methods will be demonstrated in examples. Participants can select some topics from chapters that are not covered.

Pricing and hedging without martingale measures

Docente : Professor Wolfgang Runggaldier – University of Padova
runngal@math.unipd.it
Pricing and hedging without martingale measures
Description
This part of the course complements that of Prof. Platen in the sense that it concerns some fundamental aspects of pricing and hedging that result from the following :
• In classical Mathematical finance the absence of arbitrage and the existence of an equivalent martingale measure (EMM) are regarded as essentially equivalent concepts.
• Real markets may however allow for some anomalies; in particular one may have that
a) stock price bubbles occur; in other words, the discounted price processes are strict local martingales under a risk neutral measure. This phenomenon is still consistent with classical no-arbitrage theory, but some of the classical results do not hold anymore;
b) the candidate martingale density process may be a strict local martingale and so even an EMM does not exist anymore. This is a more severe anomaly and most of the usual results from Mathematical finance then break down.
Objectives
• We shall concentrate on the second anomaly in b) and show that, even in the absence of an EMM, there is still a meaningful way to proceed in order to solve the crucial problems of pricing and hedging of contingent claims, in particular under market completeness (the latter can be defined also under absence of an EMM)
• We plan to discuss these issues along three approaches that are essentially equivalent in the sense that they all lead to the so-called real world pricing formula, which corresponds to the standard valuation formulas, where the numeraire is now the GOP and the pricing measure is the physical measure. These approaches are:
i) The "upper hedging approach". Main reference [1], see also [3].
ii) The "Benchmark approach". Main reference [4] and Chapter 3 in [5]. The equivalence between this approach and the previous one is discussed in [6].
iii) The "utility indifference" approach in the context of the Benchmark approach. Main references again [4] and Chapter 3 in [5].
• The lectures concern the above topics as outlined in the survey article [2]. Students interested in giving a seminar on these topics may have a preliminary look at Chapter 3 in [5].
1
References
[1] Fernholz, R. & Karatzas, I. (2009), Stochastic Portfolio Theory: an Overview, in: Bensoussan, A. & Zhang, Q. (eds.): Mathematical Modeling and Numerical Methods in Finance, Handbook of Numerical Analysis, vol. XV, North-Holland, Oxford.
[2] Fontana, C. (2010), Diffusion-based models for financial markets without martingale measures: an overview. Preprint.
[3] Karatzas, I. & Kardaras, K. (2007), The Num´eraire Portfolio in Semimartingale Financial Models, Finance and Stochastics, 11: 447-493.
[4] Platen, E. & Heath, D. (2006), A Benchmark Approach to Quantitative Finance, Springer, Berlin - Heidelberg.
[5] Platen, E. & Bruti-Liberati, N. (2010) Numerical Solution of Stochastic Differential Equations with Jumps in Finance. Stochastic Modelling and Applied Probability

Volume 64, Springer.
[6] Galesso, G. & Runggaldier, W.J. (2010), Pricing Without Equivalent Martingale Measures under Complete and Incomplete Observation, In: Chiarella, C. & Novikov, A. (eds.), Contemporary Quantitative Finance: Essays in Honour of Eckhard Platen, Springer.
2

Cortona 21 agosto – 2 settembre 2011

HYPERBOLIC SYSTEMS OF CONSERVATION LAWS
Stefano Bianchini - SISSA Trieste
“Systems of conservation laws : existence, uniqueness, and regularity”
Reference book : A. Bressan “Hyperbolic systems of conservation law, The one-dimensional Cauchy problem” , Oxford University Press, Oxford 2000

HYPERBOLIC SYSTEMS OF CONSERVATION LAWS
Constantine M. Dafermos - Brown University
Multidimensional hyperbolic systems of conservation laws : the little we already know, the lot that remain to be discovered .

Programmi Perugia : July 31 – September 2, 2011

ALGEBRA

Docente : Nikolai Vavilov, Università di San Pietroburgo

Programma :

ALGEBRA: INTRODUCTION TO LIE ALGEBRAS AND REPRESENTATION THEORY

Course philosophy

The idea of the course is to provide background and develop basic skills in representation theory and Lie algebras. As a preparation we start with the classical representation theory of finite groups.

Our main goal is to study two of the major pieces of the XX century Mathematics: the Cartan—Killing classification of the simple complex Lie algebras and the Cartan—Weyl classification of their finite-dimensional representations in terms of highest weights. These theories form the backbone of some central parts of Mathematics and served as models for innumerous generalisations (classification of simple algebraic groups, and their representations, classification of finite simple groups, and their representations, superalgebras, Kac—Moody algebras, infinite dimensional representations, quantum groups, etc., etc., etc.)

It is our intention to provide large portions of the proofs (maybe skipping some topological parts), develop some basic computational techniques, and convey the spirit of some most immediate applications of these results.

Time permitting we could further develop the course in the direction of representations of classical Lie groups, representations of simple algebraic groups, modular representations of finite groups, construction of Chevalley groups, or whatever other *related* subject that would be of interest to the students.

Textbooks

There are a number of excellent textbooks, some of which have equally excellent sets of exercises. Below I list two of my favourites:

1. W.Fulton, J.Harris, Representation theory: a first course. — Springer-Verlag, Berlin et al., 1991, or any later edition.
2. J.E.Humphreys, Introduction to Lie algebras and representation theory. — Springer-Verlag, Berlin et al., 1978, or any later edition.

I will also make available to the students my own notes for large parts of the course.

Below I sketch a possible detailed lecture plan for 4-5 weeks. However, if during the first week we discover that the preparation of students allows a faster pace, other topics may be covered as well.

I. Representations of finite groups

1. Linear representations, examples
2. Invariant subspaces, sub-representations and factor-representations
3. Direct sums of representations, irreducible and indecomposable representations
4. Averaging over a finite group, Maschke's theorem
5. Inner products, unitary representations
6. Schur's lemma, representations of abelian groups
7. Characters of finite groups, class functions
8. Tensor product of modules
9. Operations on characters, characters of UV и UV
10. First orthogonality relation
11. Multiplicity, Krull—Schmidt theorem
12. Decomposition of regular representation
13. The number of irreducible representations of a finite group
14. Young diagrams, representations of S_n
15. Representations of direct products
16. Second orthogonality relation
17. Character table, examples
18. Algebraic integers
19. Integrality properties of characters
20. Degrees of irreducible representations
21. Induced representations
22. Induced characters
23. Frobenius reciprocity
24. Characters of semi-direct product, Mackey's theorem

II. Lie Algebras

1. Lie Algebras, first examples
2. Classical Lie algebras and classical groups
3. Alternative algebras, Cayley—Dickson algebra, construction of G_2
4. Jordan algebras, Albert algebra, construction of F_4
5. Homomorphisms, subalgebras, ideals, characteristic ideals
6. Solvable Lie algebras
7. Nilpotent Lie algebras
8. Classification of Lie algebras of dimension ≤ 3
9. Radical, simple and semi-simple Lie algebras
10. Simplicity of sl_{l+1}
11. Representations of Lie algebras, basic constructions
12. Irreducibility and indecomposability, Weyl theorem
13. Simplicity of classical Lie algebras
14. Cartan criterion of semi-simplicity
15. Universal enveloping algebra
16. Graded and filtered algebras
17. Poincare—Birkhoff—Witt theorem and corollaries

III. Root systems and Weyl groups

1. Abstract root systems, Weyl group, first examples
2. Fundamental root systems, order and ordering
3. Height of a root, integrality
4. Length of a Weyl group element
5. System of fundamental root associated to a Weyl chamber
6. Mutual position of two roots
7. Dynkin diagrams and Coxeter graphs
8. Classification of root systems
9. Construction of classical root systems
10. Construction of G_2 and F_4
11. Construction of E_l in Minkowski space
12. Coxeter groups and real groups generated by reflections
13. Tits form and absence of cycles in the Coxeter graph
14. Fundamental inequality and its corollaries
15. Contraction of vertices
16. Inequality for the length of tails
17. Root lattice and weight lattice
18. Fundamental weights

IV. Classification of simple Lie algebras and their representations

1. Cartan subalgebras
2. Root systems with respect to a Cartan subalgebra
3. Representations of Lie algebra sl_2, construction and irreducibility
4. Highest weight and highest weight vector
5. Killing form and root subspaces
6. Fundamental sl_2 in a semisimple Lie algebra
7. Root system of a semisimple Lie algebra with respect to a Cartan subalgebra
8. Existence and uniqueness theorems, an outline of the proof
9. Structure constants in a Weyl base
10. Existence of E_l after Frenkel—Kac
11. Serre’s theorem
12. Cartan—Killing classification of simple Lie algebras
13. Modules with highest weight
14. Classification of finite-dimensional representations
15. Weyl character formula

Office hours

I will be happy to answer any questions related to the course (or otherwise) during the office hours. Outside of Russian I feel most comfortable with Italian, German and English (more or less in this order, depending on the subject).

COMPLEX ANALYSIS

Docente : Klas Diederich University Wuppertal

1 **Textbook**
As textbook we use for this course the first 4 chapters of the book by Lars Hørmander:
An introduction to Complex Analysis in Several Variables (Third Edition)
Elsevier (Amsterdam)

2 Prerequisites of the course
All students, who want to participate successfully in this course should be well acquainted with Calculus in one and several real variables, in particular, with the ideas around "Stokes theorem" and differential forms. As a preparation we recommend the book by Klaus Jähnig, Vector Analysis (Springer) or (on a much higher level) Serge Lang: Real Analysis (addison-Wesley Publ. Company).

3 Prerequisites from Complex Analysis in One complex
variable, a first quick crash course
Complex Analysis in One Complex Variable is the basis of Complex Analysis. It is the easy case. In order to understand the special features of Complex Analysis in Several Variables, we will have to present at first the basics of the theory in one Complex Variable, i.e. the case n = 1.
We will touch the following topics:
_ C1 functions are "holomorphic"
_ The open mapping theorem and the
_ The identity theorem
_ The Cauchy theory
_ Subharmonic functions
1
4 Cauchy integrals on polydiscs
Polydics are direct products of discs. The theory of holomorphic functions on them can mostly be understood by dealing with one variable after the other. The main topics of this first main section of the course arise from considering
1. Cauchy integrals on polydiscs and the resulting Cauchy inequalities
and by studying
2. the inhomogeneous Cauchy-Riemann equations in a polydisc
This leads in particular to the
Theorem 4.1 (The Poincaré lemma for the @-operator on polydiscs) Let D be an open polydisc and let f 2 C1(p;q+1)(D) satisfy the condition @f = 0. If D0 is relatively compact in D we can find a u 2 C1(p;q)(D0) with @u = f in D0.
A side-aspect of this will lead us to considering power series and Reinhardt domains.
5 The Cartan-Thullen theory of holomorphic convexity
With this chapter we enter into the heart of Complex Analysis showing us the first very specific features of the theory in several (n > 1) complex variables.
We come to
domains of holomorphy: their interior and exterior characteristic properties.
The notion of hulls and their properties. We discover the basic properties of plurisubharmonicity and pseudoconvexity.
This leads to the

Theorem 5.1 (First main theorem: Cartan-Thullen) Domains of holomorphy are pseudoconvex.

For a long time it has been an outstanding problem of Complex Analysis, which even today is still unsolved in some of its tricky versions, allthough it has been solved by K.Oka and H. Grauert in the form, how we can ask it at the moment:

Question 5.2 (The Levi Problem) Is any peudoconvex domain in Cn a domain of holomorphy?

It will be the major goal of this course to work out a solution to the Levi problem by mean of the L2-theory of the @-operator.

2

6 A first consideration of Runge domains in Cn

Domains of holomorphy D with the property that the set of polynomials on them are dense in their algebra of holomorphic functions enjoy a particularly strong global convexity condition. They are much more easy to deal with under the point of view of the Levi problem than arbitrary pseudoconvex domains in Cn. In particular the equations @u = f can be globally and much more easily solved on them. This will be done in this section.

7 L2-estimates and existence theorems for the @ operator

7.1 Linear, unbounded, densely defined operators between Hilbert spaces

Definition 7.1 1. The Hilbert spaces L2
(p;q)(; ') and their norms and L2
(p;q)(; loc)

2. The spaces D(p;q)() as dense subspaces

Definition 7.2 1. Linear, closed, densely defined operators T : H1 ! H2 between Hilbert spaces

2. Their Ranges and Domains

3. The Hilbert space adjoint T_ of T : H1 ! H2

Lemma 7.3 The identity T__ = T and other relations between T and T_

We refer all readers who want to study this tool of functional analysis more deeply to the excellent presentation in the book "Functional Analysis" by Walter Rudin (McGraw-Hill Book Company, New York 1973)

An abstract, basic, functional-analytic existence theorem as a consequence of the Hahn-Banach theorem:

Lemma 7.4 F = RT if and only if
kfkh2 _ CkT_fkH18f 2 F \ DT_ (7.1)

From this we obtain a kind of dual statement, which will be the basis for proving aproximation theorems

Lemma 7.5 Let T : H1 ! H2 be a closed densely defined operator and F _ H2 a closed subspace with RT _ F. Let (7.1) hold true. Then for every v 2 H1, v 2 ker(T)? one can find f 2 DT_ such that T_f = v and
kfkH2 _ CkvkH1 (7.2)

3

In order to be able to apply these abstract lemmas in a more concrete situation, we need criteria for the density of D(p;q+1) in certain subspaces with respect to the graph norm given by
f 7! kfk'2 + kT_fk'1 + kSfk'3

The exact situation will be descrbed in

Lemma 7.6 Density of D(p;q+1)() with respect to the graph norm

Another important tool is the study of

Lemma 7.7 (existence of smoothings)

7.2 A first existence theorem for @ on pseudoconvex open sets in Cn

We are now able by some rather carefull analysis to solve the @-equations in the sense of distribution theory on any open pseudoconvex set in Cn. Our main theorem will be

Theorem 7.8 (Solving @u = f for all (p; q + 1) forms) Let be a pseudoconvex open set in Cn. Then the equation @u = f has (in the sense of dstribution theory) a solution u 2 L2
(p;q)(; loc) for every f 2 L2
(p;q+1)(; loc) such that @f = 0

Next, we need interior regularity properties in the sense of the Sobolev spacesWs
(p;q+1)(; loc).

Our main result will be

Theorem 7.9 (regularity in Sobolev norms) Let be a pseudoconvex open set in Cn and let 0 _ s _ 1. Then the equation @u = f has a solution u 2 Ws+1
(p;q)(; loc) for
every f 2 Ws
(p;q+1)(; loc) such that @f = 0. Every solution of the equation @u = f has this property when q = 0.

8 The solution of the Levi problem for pseudoconvex
domains in Cn

It is obvious, that each open set in C1 is pseudoconvex and,in fact, also a domain of holomorphy. This important observation will give us the chance, to prove by induction over n, that each pseudoconvex domain in Cn for n arbitrary is a domain of holomorphy. This means, that we want to prove by induction over n, that one has:

Theorem 8.1 An open set in Cn is a domain of holomorphy if it is peudoconvex.

This is the claimed inverse of the Cartan-Thullen theorem. We will prove it by solving the @-equation for all bidegrees (p; q). We formulate this in:

Theorem 8.2 Let be an open set in Cn, such that the equation @u = f has a solution u 2 C1(0;q)() for every f 2 C1(0;q+1)() such that @f = 0 (q = 0; : : : ; n Ÿ 2). Then is a domain of holomorphy.

4

9 Approximation theorems

As we indicated already earlier the dualization of Lemma 7.4 with inequality 7.1 leads to an approximation result. The first technical statement of this is

Lemma 9.1 Let p be a strictly plurisubharmonic C1 function in such that
Kc = fz : z 2 ; p(z) _ cg __ for every c 2 R (9.1)
Then every function analytic in a neighborhood of K0 can be approximated in L2-norm over K0 by functions in O().

We can reformulate this statement as

Theorem 9.2 (Aproximation on plurisubharmonically convex hulls) Let be a pseudoconvex open set in Cn and K a compact subset of such that ^K P
 = K. Then
every function which is analytic in a neighborhood of K can be approximated uniformly on K by functions in O()

The final form, into which we want to bring our approximation results, will be

Theorem 9.3 Let 1 _ 2 be domains of holomorphy. Then the following conditions are equivalent:

1. Every function in O(1) can be approximated by functions in O(2) uniformly on every compact subset of 1 (1 is then called a Runge domain relative to 2)
2. For every compact set K _ 1 we have ^K2 = ^K1
3. For every compact set K _ 1 we have ^K2 \ 1 = ^K1
4. For every compact subset K _ 1 we have ^K2 \ 1 __ 1

10 Supplement (If time allows:) Existence theorems in

L2-spaces

Estimates of weighted norms have deep consequences in the L2-theory. If time allows we will enter into this domain by proving a rather strong such estimate based on the following technical

Lemma 10.1 Let be a pseudoconvex open set in Cn and let ' be a real valued function in C2() such that

c
n X1
jwj j2 _
n Xj;k=1
@2'
@zj@_zk
wj _ wk (10.1)

where c is a positive continuous function in . If g 2 L2
(p;q+1)(; ') and @g = 0 it follows
that one can find a u 2 L2
(p;q)(; ') with @u = g and
Z juj2 e'd_ _ 2 Z jgj2 e'=cd_ (10.2)
provided that the right hand side is finite.

5

We will deduce from this:

Theorem 10.2 Let be a pseudoconvex open set in Cn and ' any plurisubharmonic function in . For every g 2 L2
(p;q+1)(; ') with @g = 0 there is a solution u 2
L2
(p;q)(; loc) of the equation @u = g such that
Z juj2 e'(1 + jzj2)2d_ _ Z jgj2 e'd_ (10.3)

Remark 10.3 The essential new feature in this theorem is, that there is no smoothness condition on '.

Lectures in English

FUNCTIONAL ANALYSIS

Docenti : Yuli Eidelman and Vitali Milman University of Tel Aviv
General Program:

(a) Introduction (covering chapters 1-4, assuming much is known in advance)
(b) Fundamental Theorems of Functional Analysis (Chapter 9)
(c) Fredholm Theory for compact operators (Chapter 5)
(d) Specrtal Theory of Self adjoint operators (Chapter 6)
(e) Functions of operators and Spectral decomposition (Chapter 7)
If time permits:
(f) Banach Algebras (Chapter 10).
(g) Introduction to local theory of normed spaces (from the book of Milman and Schechtman "Asymptotic theory of finite dimensional normed spaces")
The ordering may also be (closer to the way the book is written): (a) (c) (d) (e) (b) (f)
The question whether we will get to (f) and (g) relies mainly on whether most of (a) can be assumed to be known or not.
Lectures in English

Testbook : Eidelman, Milman and Tsolomitis : "Functional Analysis: An Introduction".

DIFFERENTIAL EQUATIONS IN MATHEMATICAL PHYSICS

Docente : Russell Johnson, Univ. di Firenze

Programma

The second part of the course is intended as an introduction to the study of the "algebro-geometric" solutions of the Korteweg - de Vries equation, a nonlinear PDE which describes the motion of shallow water waves under certain conditions.

The starting point is the remarkable observation of Gardner-Green-Kruskal-Miura that the Korteweg – de Vries equation can be derived by deforming the potential of the one-dimensional Schroedinger operator in an isospectral way. This leads, of course, to the study of isospectral classes for the one-dimensional Schroedinger operator.

The following points will be discussed. –

1) The classical spectral theory of the 1-D Schroedinger operator (finite interval, half-line, full line)
2) The Bebutov approach to nonautonomous differential equations
3) The "nonautonomous" approach to the study of the spectral theory of the 1-D Schroedinger operator
4) The algebro-geometric Schroedinger potentials
5) The algebro-geometric solutions of the K–dV equation.

Text: E. Coddington and N. Levinson, "Theory of Ordinary Differential Equations", McGraw-Hill, 1955 and 1984.

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Docente : Carlo Domenico Pagani, Politecnico di Milano

Programma

La prima parte del corso consiste di 8 lezioni aventi per oggetto:
Non linear first order PDE:
1. First order PDE, boundary problem
2.The method of characteristics, local existence theorem
3.Hamilton-Jacobi equations, weak solutions,uniqueness
4.Conservation laws, weak solutions, uniqueness.

Il testo di riferimento è:
L. Evans, Partial Differential Equations,
Graduate Studies in Mathematics, 19; Chp. 3 : Non linear first order PDE.

ALGEBRAIC GEOMETRY

Docente : Enrique Arrondo, University Complutense of Madrid)

The goal of the course is to provide a fast introduction to the geometry of projective varieties, based mostly on examples. Being optimistic, the items could be organized in the following weekly distribution:

1. Introduction to projective sets: graded ring, Nullstellensatz, Hilbert polynomial, invariants.
2. Morphisms of projective sets: products, projections.
3. Parameter spaces: Grassmannians, Fano varieties, Hilbert spaces.
4. Counting degrees and dimension of classical examples.
5. Special varieties: minimal degree, secant and dual varieties, open problems.

Text: The spirit and most of the examples are extracted from the textbook:

J. Harris, *Algebraic Geometry: A first Course*, Graduate Texts in Mathematics, Springer 1992.

The teacher will also use his own notes, where most of the complete details are done. They are available at www.mat.ucm.es/~arrondo/projvar.pdf (just sections 1-13).

Prerequisites: Only familiarity with projective geometry and the use of ideals in algebra will be required to follow the course.

Language: English.

Nota per gli studenti di lingua italiana: Anche se il corso si svolgerà interamente in inglese (salvo nell'improbabile caso che tutti gli studenti sappiano l'italiano), il ricevimento studenti e gli esami si potranno fare in lingua italiana.

PROBABILITY

Docente : S. L. Zabell, Northwestern University

Syllabus: The course will follow the text closely. Homework will be assigned mainly from the problems at the end of sections in the book, but occasionally exercises from other sources may be assigned. The exact coverage will depend on the background of the class (in particular, prior knowledge of measure theory). In any case, we will aim at covering at least the material in the first nine sections of Billingsley.

Lectures in English

Prerequisites: Rigorous calculus (sup, inf, limsup, liminf, continuity, uniform continuity, uniform convergence, etc.); linear algebra; modern analysis (metric spaces, Hilbert spaces). Familiarity with elementary probability and Lebesgue measure is desirable.

Useful background reading: William Feller, *An Introduction to Probability Theory and its Applications*, Vol. I, 3rd edition, Wiley, 1968, Chapters 1—2, 5—6

Text book : Patrick Billingsley, Probability and Measure, 3rd edition, Wiley 1995

Additional reference: John Lamperti, *Probability*, 2nd ed., Wiley, 1996.